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AN IMAGE REGISTRATION ALGORITHM USING SAMPLED BINARY CORRELATION

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ABSTRACT

One of the problems associated with the automatic image processing of satellite photographs such as weather maps is the need for image registration; that is, the fitting of a map that has some translational and rotational bias to a known data base. This paper investigates a least square method of image registration using an image that has been converted into a boundary map with a pixel representation of 1 for land, -1 for water and zero for cloud pixels. A sampled correlation array is constructed about the correlation peak of the binary cross-correlation for the coded satellite map against its data base by shifting the satellite map to locations on a given grid, and performing an accumulation of the pixel-by-pixel comparisons between the satellite image and its data base over the whole map or a smaller search window. A least square approximation of the translational and rotational bias can then be performed using the data from this sampled correlation array, fitted against a shape such as an elliptical cone.

I. INTRODUCTION

One of the problems associated with the automatic image processing of incoming satellite maps such as weather map photographs, military reconnaissance photographs, or earth resource maps is the need for image registration; that is, the fitting of a map that has some translational and rotational bias to a known data base. (1) The common method for resolving registration translation error is cross-correlation. Cross-correlation is typically computed by means of the Fast Fourier Transform algorithm. (2,3)

This paper presents a method for resolving rotation and arbitrary translation of an unregistered satellite map. The method is based on the following procedures.

1. The data base map's pixels are converted into single bits representing two classification regions.

2. The misregistered map's pixels are converted into a two bit code representing a two region classification and one indeterminate or noise classification.

3. A sampled binary cross-correlation of the noisy satellite data is generated by adding plus one to the correlation sum for matched pixels, zero to the correlation sum for noise pixels, and negative one to the correlation sum for mismatched pixels.

4. The sampled binary cross-correlation consists of a large two-dimensional array, from which a small array is taken about the array's maximum correlation value.

5. A least squares fit is performed on the small array to a simplified model of the expected cross-correlation function.

6. The results of the least squared calculation produce the rotational and translational correction to the misregistered map.

II. CROSS-CORRELATION

It is recognized that the bottleneck to fast automatic image registration is the cross-correlation computational requirements. The amount of computation required for a square map, N on a side is proportional to N^P, where $1 < P \leq 2$, and also proportional to I^P, where I is the maximum allowed translation.

The computational procedure in this paper reduces the cross-correlation computation by two factors. One factor is due to sampling and hence is the ratio $(I/K)^P$, where K is the sampling interval in units of pixels. The second factor is

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due to the reduction in calculation complexity (i.e. going from a FFT to a binary correlation) leading to a reduction factor from three to ten. (4)

These substantial savings in computational requirements are not clearly applicable to arbitrary maps. The sampling requires that the dimension of the major binary classified regions be large compared to the sampling interval, for example, a map which contains large regions of land and water. One might be tempted to discount the sampling concept as having limited value when dealing with apparent salt and pepper maps. However, many such maps may still be classified by preprocessing/filtering such that region "i" is salt and pepper of spatial frequency distribution "i".

III. APPROACH

The weather map (this paper use a weather picture as the misregistered image) is stored in the computer's memory as a three level map containing two component levels (land and sea) with values of 1 and -1 and a disturbance level (clouds) with a value of zero. The image is assumed to be free of geometric distortions and magnification error with respect to a reference image stored in a data base. The coordinate systems of both the weather map and data base map have their origins located at the center of the map. The coordinates of the weather map (u',v') are related to the data base coordinated (u,v) by the linear transformation.

$$u' = (u+\alpha)\cos\theta-(v+\beta)\sin\theta \qquad (1)$$

$$v' = (u+\alpha)\sin\theta+(v+\beta)\cos\theta \qquad (2)$$

where α and β is the translational misregistration along u and v respectively and θ is the angular rotational misregistration of the weather map image.

A sampled two-dimensional correlation is performed between the weather map image and the data base. This is done by shifting the weather map along the u- and v-axes in equal-distant increments and performing an accumulation of the pixel-by-pixel comparisons over the whole picture or search window at each shift point. This comparison is defined as

$$R(u,v) = \sum_{j=-J}^{J}\sum_{k=-K}^{K} M_w(j-u,k-v)\, M_d(j,k) \qquad (3)$$

where j and k are pixel indices in a 2J by 2K search window area; M_w and M_d are the weather map and data base map respectively.

Whenever like pixels are compared, 1 is added to the accumulation and whenever unlike pixels are compared, -1 is added to the accumulation. If a cloud pixel is detected, the accumulation is left unchanged, since clouds represent unusable portions of the weather map for purposes of registration and are therefore masked out of the summation.

The sampled correlation is then searched to locate the peak sample. This peak sample is separated from the continuous correlation peak by less than half the sample grid in u and less than half the sample grid in v.

A square sampled correlation array is obtained from a set of samples around and including the peak sample. The peak sample serves as the center element of the array. For example, a 5 x 5 correlation array is constructed by taking the peak sample and all samples within ±2 samples of the peak in both the u- and v-directions.

Once the sampled correlation array has been established, the sample elements of the array and their corresponding coordinates are entered into a least squares approximation to estimate the translational and rotational bias on the weather map. This translational and rotational result will be with respect to the location of the peak sample. Any translational bias on the picture will be no more than half the sample grid in u or v since the array is centered about the sampled correlation peak.

IV. LEAST SQUARE REGISTRATION METHOD

The least square approximation method is used to fit physical data to a chosen function. A cone with an elliptical base could be chosen as the model with which one can fit the physical correlation data using least squares. Since the map is to be registered in translation as well as rotation, the cone equation,

$$[h(x,y)]^2 = [x-\alpha]^2/a^2+[y-\beta]^2/b^2 \qquad (4)$$

where α and β are the u- and v-directional translations respectively, is rotated in θ yielding

$$[h(x,y)]^2 = [x\cos\theta-y\sin\theta-\alpha]^2/a^2 + [x\sin\theta+y\cos\theta-\beta]^2/b^2 \qquad (5)$$

After some algebraic manipulation the cone equation becomes

$$\begin{aligned}[h(x,y)]^2 = {} & [\cos^2\theta/a^2+\sin^2\theta/b^2]x^2 \\ & +[\sin^2\theta/a^2+\cos^2\theta/b^2]y^2 \\ & +2\sin\theta\cos\theta[1/b^2-1/a^2]xy \\ & -2[\alpha\cos\theta/a^2+\beta\sin\theta/b^2]x \\ & +2[\alpha\sin\theta/a^2-\beta\cos\theta/b^2]y-[\alpha^2/a^2+\beta^2/b^2]\end{aligned} \qquad (6)$$

or

$$[h(x,y)]^2 = t_1x^2+t_2y^2+t_3xy+t_4x+t_5y+t_6 \qquad (7)$$

The coefficients, t_n, which are obtained by means of least squares are then used to determine α and β, the translational shift, and Θ, the rotational component [see appendix].

To simulate this method of picture registration, a hypothetical map model was used (figure 1). The 5 x 5 sampled correlation array was constructed centered about the origin of the data base map with a grid spacing of 20 pixels. A translational and/or rotational bias was introduced to the weather map model with the translation being within ±10 pixels in either the u- or v-direction. A rotational bias of up to 22.5 degrees was also introduced to the map model.

The normalized elements of the 5 x 5 sampled correlation array, the (u,v) coordinates of the sample elements and the coefficients of equation (7) yield a set of 25 equations in 6 unknowns, of the form

$$[h^2] = [u^2, v^2, uv, u, v, 1]\begin{bmatrix} t_1 \\ t_2 \\ t_3 \\ t_4 \\ t_5 \\ t_6 \end{bmatrix} \qquad (8)$$

or

$$[h^2] = \begin{bmatrix} 1600 & 1600 & 1600 & -40 & -40 & 1 \\ 400 & 1600 & 800 & -20 & -40 & 1 \\ & & \vdots & & & \\ 400 & 1600 & 800 & 20 & 40 & 1 \\ 1600 & 1600 & 1600 & 40 & 40 & 1 \end{bmatrix}\begin{bmatrix} t_1 \\ t_2 \\ t_3 \\ t_4 \\ t_5 \\ t_6 \end{bmatrix} \qquad (9)$$

where h^2 is defined as the normalized correlation element,

$$h^2 = [H-R(u,v)]^2/H^2 \qquad (10)$$

where H is the normalization constant.

Equations (8) and (9) form the matrix equation

$$[h^2] = w = At \qquad (11)$$

which is solved by

$$A^Tw = [A^TA]t \qquad (12)$$

$$[A^TA]^{-1}A^Tw = [A^TA]^{-1}[A^TA]t \qquad (13)$$

$$[A^TA]^{-1}A^Tw = t \quad , \qquad (14)$$

the least square solution to t (5).

V. RESULTS(6)

Using the hypothetical weather map model shown in figure 1, sampled correlation arrays were generated with weather map translational biases of 0, ±2.6, ±5.2 pixels for α and β and rotational biases between 0 and 22.5 degrees. These values were combined to provide cases for examination with or without cloud noise. The elements of these correlation arrays were entered into the least square method and solutions were obtained for the translational and rotational biases originally applied to the weather map model. Table 1 lists the originally applied biases versus their solutions for translated and rotated maps containing no clouds. A relative error (in percent) is also given of the least square solutions with respect to the applied biases.

When one examines case 1 of table 1, the case of zero shift and rotation, it can be seen that no zero bias error exists in this procedure. In other words, with zero shift and rotation, the least square method yielded an exact solution offering no bias of its own.

Cases 2 through 6 show the results of placing only rotation on the maps. Table 1 illustrates that, in these cases, when no translation has been introduced, the least square method gave an exact result for the zero translational bias. However an average error in Θ of approximately 24% was introduced to the rotational result.

Cases 7 through 10 show the effect of a pure translational bias in different quadrants. The solution of pure translation showed average errors of ±8.7% in the u-direction and approximately 16.7% in the v-direction. When translations were applied to reflected points in other quadrants there was no change in the results. This is seen by inspection of case 7 vs. case 10 and case 8 vs. case 9. A small rotation error was introduced, however, by the approximation of these nonrotational cases.

A study of the effect of normalization was made on cloudless maps with rotations of 22.5 degrees or less. In figures 2, 3 and 4, a plot of the least square solutions versus normalizations from 100,000 to 7,500,000 show that a pole exists at some point within this interval and from there the solution slopes asymptotically toward a solution that is within ±7% of half the desired value. This tends to indicate that a faster solution may be achieved by doubling the average least square solution using extreme high and low normalization points.

The next investigation regarded the introduction of cloud noise onto the map. A cloud of 100 x 100 pixels was placed on the map centered at the northwest corner of the island as shown in figure 1. At this point, the cloud's effect will be felt during the correlation of both land and sea pixels and since it is a corner, it will affect the correlation in the northwest differently than in the southeast. Table 2 gives the least square solutions for clouded maps.

The first case to be examined is case 15 where no bias was applied to the weather map. In this case a bias does result due to the cloud noise and therefore it could be expected that a map with cloud noise could never be registered exactly.

A comparison of cases 16, 18, 19 and 20, with cases 4, 12, 13 and 14 of table 1, shows that the cloud noise had little effect on the rotational result. However, when the rotational bias approached zero, as seen in case 15 and 17, a rotation error of as much as one degree was introduced by the cloud (the respective cases, 1 and 7, of table 1 produced no more than 0.155 degree of rotation error). It is interesting to note that, in case 20, the cloud had no affect on the translation or rotation error when compared to case 14. This result is due to the fact that, the large rotation placed the cloud entirely over the water area of the data base causing little change in the correlation array.

VI. ITERATIVE REGISTRATION METHOD

From the previous section, it was apparent that a single, one-pass registration using least squares may not be sufficient for proper image registration. With the decreasing cost of random access memory and processing hardware, the iterative approach becomes more economical. In this method, the procedure outlined in this paper would be performed by loading the weather map in one page memory and performing the correlation with the data base in another page of memory. The shifting operation necessary to obtain the correlation array may be incorporated in the address of the data base pixel being used for comparison. Once the sampled correlation array has been constructed and a least square solution for the translation and rotation has been determined, the weather map would then be reloaded with an opposite shift and rotation bias applied to it as it is placed in memory from a mass storage device such as a disc drive and another least square registration would be performed on this corrected map.

For example, if a weather map that has rotation only, such as case 6 ($\alpha=\beta=0, \Theta=22.5$), is applied to this registration method, then the rotation will have been determined to be (from table 1) 16.4 degrees. The weather map is then reloaded into memory with a -16.4 degree rotational bias added to it changing the actual rotational bias on the weather map to

$$22.5 - 16.4 = 6.1 \text{ degrees.}$$

This new bias is close to case 3 ($\alpha=\beta=0$, $\Theta=5$ degrees) which produces a least square estimate (from table 1) of 3.873 degrees. If the weather map is corrected by this new estimate then the new actual rotational bias would be

$$6.1 - 3.873 = 2.227 \text{ degrees,}$$

which is close to case 2 ($\alpha=\beta=0$, $\Theta=2.5$ degrees). Another repetition of this process yields an estimate of 1.448 degrees and when the map is corrected by this amount, the actual rotational bias would be

$$2.227 - 1.448 = 0.779 \text{ degrees.}$$

Now after three passes, the rotational error with respect to the original bias is

$$\frac{0.779}{22.5} \times 100 = 3.5\%$$

which is considerably less than the error of case 6 in table 1 of 27.1%.

The size of memory needed to perform this type of registration would depend on the search window size and the size of the data base. The data base area would have to be large enough to accomplish the shift of the weather map when performing a correlation. Two bits per pixel would be needed to represent land, sea and cloud pixel values.

VII. CONCLUSION

This paper demonstrates that a sampled binary correlation technique can be effectively used to calculate misregistration displacement and rotation. The least squares misregistration is approximate, sine the correlation surface is approximated as an elliptic cone rather than the auto correlation function. If one employes the actual auto correlation function rather than a simple elliptic cone to compute misregistration, then one anticipates negligible computation error.

The least squares method itself requires very little computer time, although much time and/or hardware is needed to generate the correlation arrays used by this method. Since this approximation can be improved by either using an iterative method or by using renormalization and the sampled correlation computation time can be improved with an array processor, these ideas can be combined to speed up the registration process compared to FFT (2,3) or other techniques (1).

There is a need for additional investigation into the problem of cloud noise on the misregistered image. Cloud noise caused an increased amount of error as compared to the results for cases without any clouds. It was shown that the presence of a cloud made it impossible to achieve an exact solution because the cloud introduced a zero bias error when no translation or rotation had been applied. Future studies into methods of reducing the effects of cloud noise are needed before the least squares registration method could be applied to applications such as weather analysis.

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CASE #	ORIGINAL BIAS α	β	θ	LEAST SQUARE RESULTS α	|α| (%) ERROR	β	|β| (%) ERROR	θ	|θ| (%) ERROR
1	0	0	0	0	0	0	0	0	0
2	0	0	2.5	0	0	0	0	1.448	-42.1
3	0	0	5.0	0	0	0	0	3.873	-22.5
4	0	0	7.5	0	0	0	0	6.366	-15.1
5	0	0	10.0	0	0	0	0	8.603	-14.0
6	0	0	22.5	0	0	0	0	16.400	-27.1
7	5.2	2.6	0	5.001	-3.8	2.317	-10.9	0.155	-
8	2.6	5.2	0	2.952	13.5	4.034	-22.4	0.142	-
9	2.6	-5.2	0	2.952	13.5	-4.034	-22.4	0.142	-
10	-5.2	2.6	0	-5.001	-3.8	2.315	-10.9	0.155	-
11	5.2	2.6	2.5	5.634	8.3	1.822	-29.9	1.617	-35.3
12	5.2	2.6	7.5	7.090	36.3	1.509	-42.0	6.660	-11.2
13	2.6	5.2	7.5	4.166	60.2	4.134	-20.3	6.883	-8.2
14	5.2	2.6	22.5	7.125	37.0	0.462	-82.2	16.284	-27.6

Table 1. Least square results for noncloud maps

CASE #	ORIGINAL BIAS α	β	θ	LEAST SQUARE RESULTS α	|α| (%) ERROR	β	|β| (%) ERROR	θ	|θ| (%) ERROR
15	0	0	0	-0.241	-	1.224	-	0.729	-
16	0	0	7.5	0.522	-	0.172	-	6.506	-13.3
17	5.2	2.6	0	4.809	-7.5	3.648	40.3	1.080	-
18	5.2	2.6	7.5	7.503	44.3	1.748	-32.8	7.772	-9.7
19	2.6	5.2	7.5	4.626	77.9	4.385	-15.7	6.832	-8.9
20	5.2	2.6	22.5	7.125	37.0	0.462	-82.2	16.284	-27.6

Table 2. Least square results with cloud noise

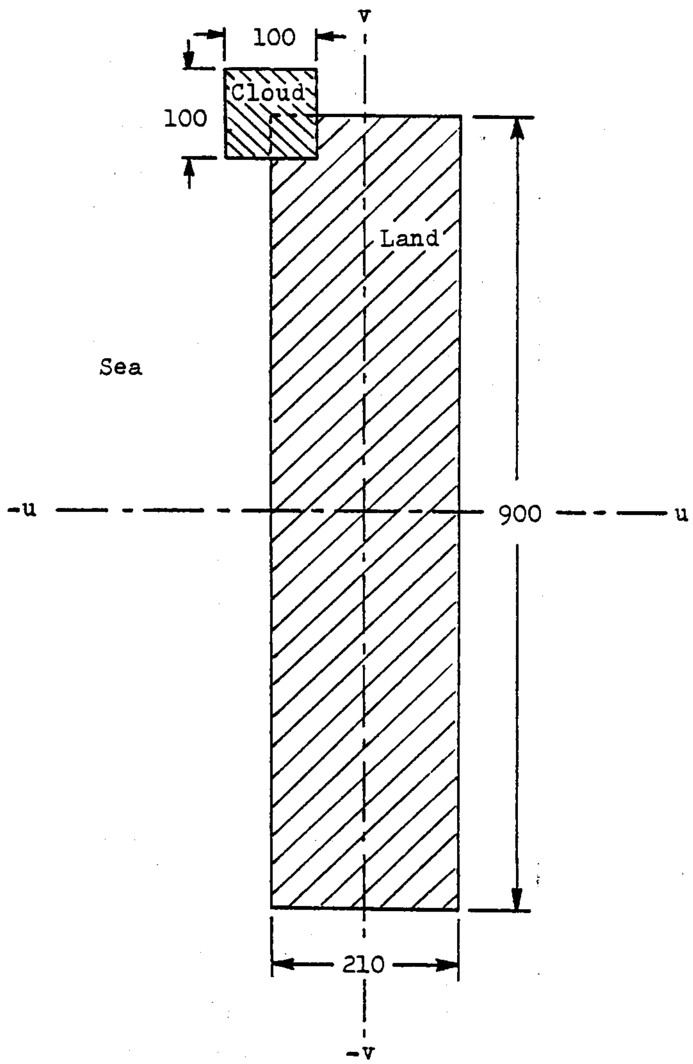

Figure 1. Hypothetical map model

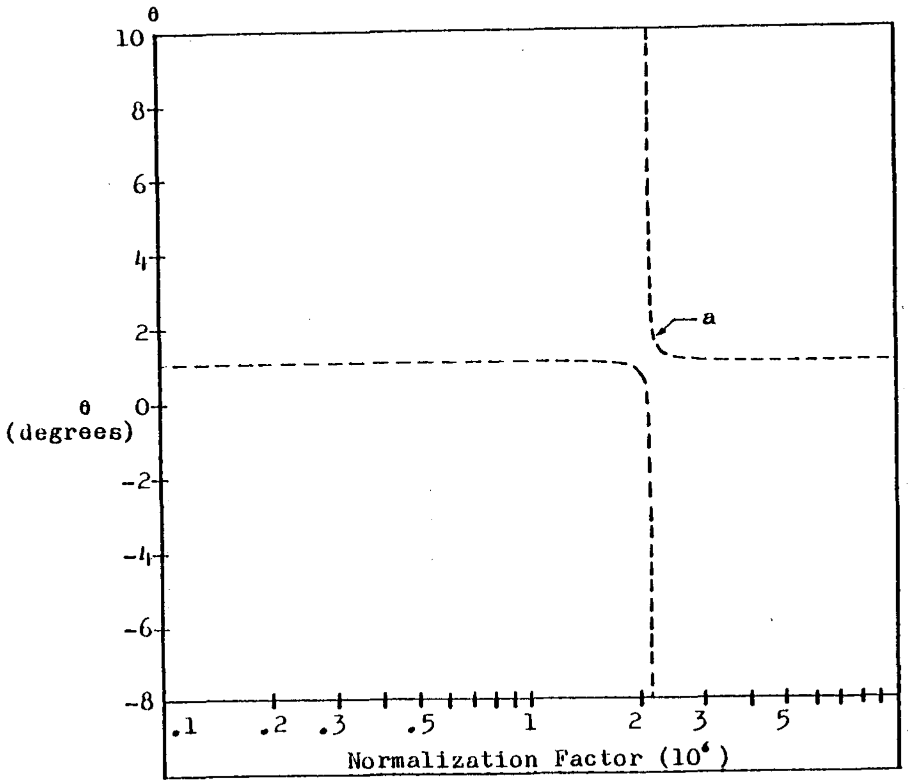

Figure 2. Normalization of Θ = 2.5°

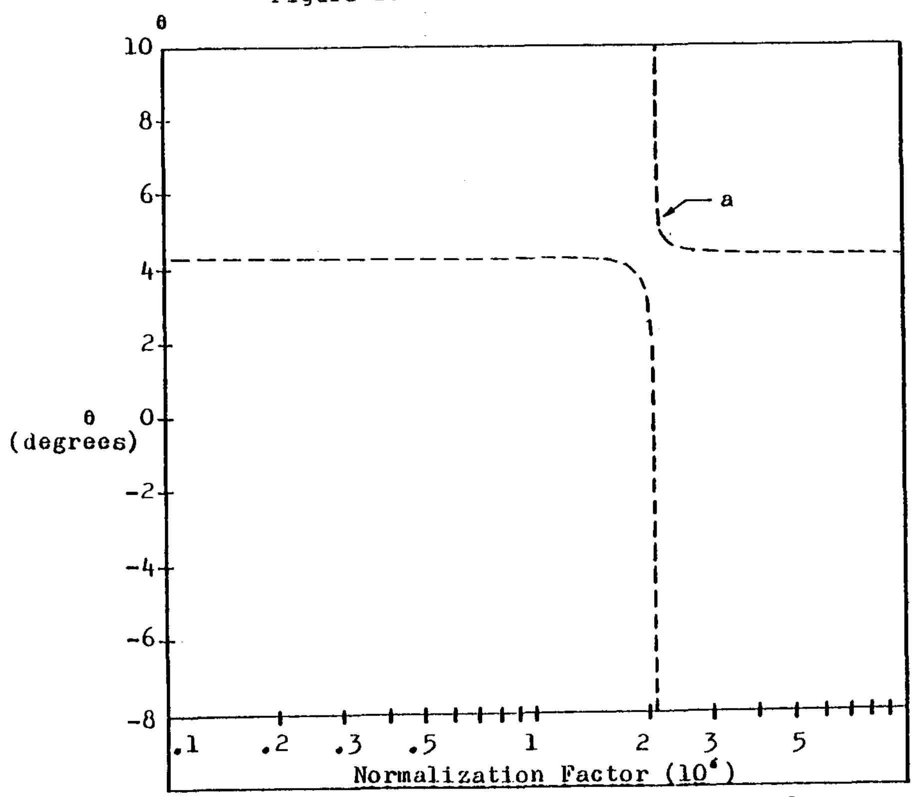

Figure 3. Normalization of Θ = 7.5°

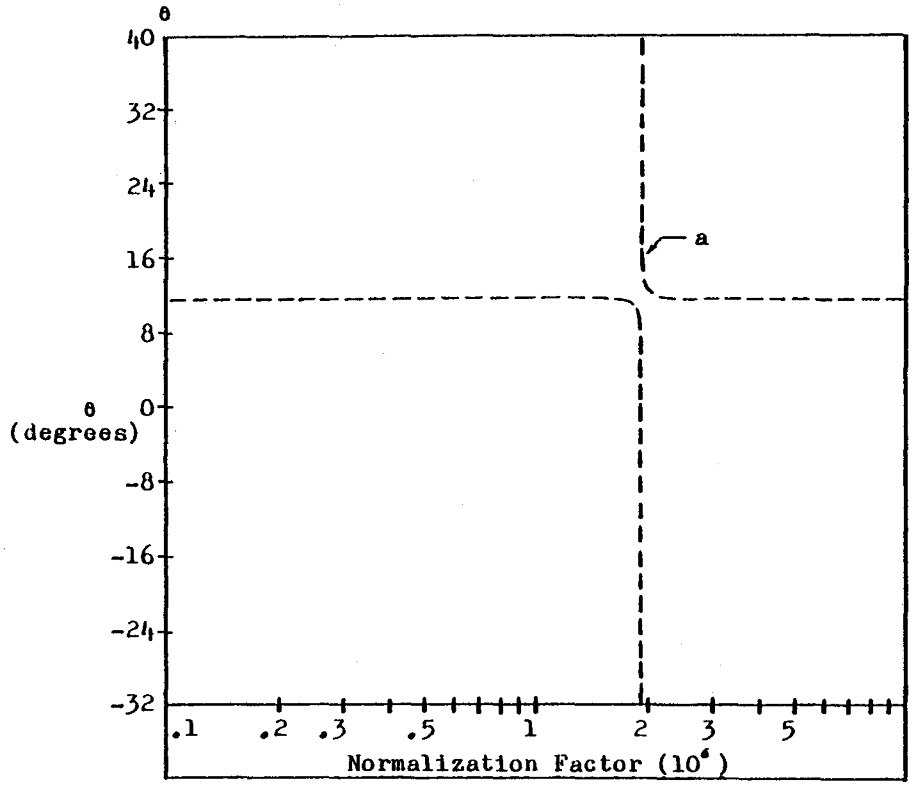

Figure 4. Normalization of $\theta = 22.5^{\circ}$

APPENDIX. SOLUTION OF REGISTRATION PARAMETERS

The coefficients of equations (6) and (7) of the main text can be written as the following set of equations:

$$t_1 = \cos^2\theta/a^2+\sin^2\theta/b^2 \quad \text{(A.1)}$$

$$t_2 = \sin^2\theta/a^2+\cos^2\theta/b^2 \quad \text{(A.2)}$$

$$t_3 = 2\sin\theta\cos\theta[1/a^2-1/b^2] \quad \text{(A.3)}$$

$$t_4 = -2[\beta\sin\theta/b^2+\alpha\cos\theta/a^2] \quad \text{(A.4)}$$

$$t_5 = 2[\alpha\sin\theta/a^2-\beta\cos\theta/b^2] \quad \text{(A.5)}$$

$$t_6 = \alpha^2/a^2+\beta^2/b^2 \quad \text{(A.6)}$$

The coefficients t_1, t_2 and t_3 can be combined,

$$t_3/(t_2-t_1) = \tan 2\theta \quad \text{(A.7)}$$

to yield a value for the rotation,

$$= 1/2 \tan^{-1}[t_3/(t_2-t_1)] \quad \text{(A.8)}$$

For convenience intermediate values of F and G are defined as

$$F = (t_3/\sin 2\theta)+(t_1+t_2) = 2/b^2 \quad \text{(A.9)}$$

$$G = (t_3/\sin 2\theta)-(t_1+t_2) = -2/a^2 \quad \text{(A.10)}$$

which leads to the solution of the elliptical values,

$$a^2 = -2/G \quad \text{(A.11)}$$

$$b^2 = 2/F \quad . \quad \text{(A.12)}$$

These values along with the coefficients t_4 and t_5, yield the translational components,

$$= -(t_4\sin\theta+t_5\cos\theta)/F \quad \text{(A.11)}$$

$$= (t_4+\beta F\sin\theta)/G\cos\theta \quad . \quad \text{(A.12)}$$